

# THE SOLAR SEMIDIURNAL PRESSURE WAVE OVER NORTH AMERICA

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## ABSTRACT

A geographical representation of the amplitude and phase distribution of the solar semidiurnal pressure oscillation over North America shows clearly the amphidromic point expected here because of the superposition of the traveling and standing semidiurnal waves. It indicates also certain peculiarities in the phase and amplitude distribution which may be due to orographic and coastal influences. The seasonal variations of the solar semidiurnal pressure wave for the western stations are quite different from those for the stations in the central plains and in the east of the continent. Especially the total phase change throughout the year is much smaller in the west.

## 1. INTRODUCTION

The solar semidiurnal pressure wave (denoted in the following by  $S_2$ ) is one of the most regular meteorological phenomena. Earlier work by A. Schmidt [5] and others, and later harmonic analyses and geographical representations by Simpson [6] and Haurwitz [2] have shown that its global distribution can be described very adequately by the superposition of two oscillations. One of these travels around the earth with the sun. It has its largest amplitude, about 1 mb., at the equator and decreases toward both poles. Its maximum occurs on the average for the earth around 9 hr. 45 min. Mean Local Time. The other vibration is a standing oscillation with zonal symmetry. It is important only at high latitudes, where the traveling wave is small, and its maximum amplitude is here about 0.1 mb. Its maximum at high latitudes is reached on the average at about 11 hr. 30 min. GMT. According to theory the phase of the standing oscillation should be reversed between approximately 30° N. and S. latitude, but it cannot be determined here from observations because of the much larger traveling oscillation.

Despite its great regularity  $S_2$  shows certain peculiarities in its distribution which require further study. The present paper is concerned with the characteristics of  $S_2$  over North America. It is in many respects a sequel and a supplement to an investigation by Spar [7] which deals with  $S_2$  in the United States.

determine  $S_2$ . They form the basis of the present investigation together with data for a few additional stations in northern Canada found elsewhere (Haurwitz [2]). The stations available for Canada and Alaska are indicated by dots on figure 1.

Since the period of observation is 10 yr., it can be expected that the computed amplitudes and phases are sufficiently accurate, although it is not possible to determine the radii of the probable error circles from the published data. To give some indication of the magnitude of these radii, table 1 shows determinations of  $S_2$  for six stations in northern Canada. Four of these are from another investigation (Haurwitz and Sepúlveda [3]), and the two most northerly stations have not been published before. In this table  $A_2$  denotes the amplitude,  $\alpha_2$  the phase angle, so that

$$S_2 = A_2 \sin(30t + \alpha_2)$$

where  $t$  is Local Mean Time. The number of years available for each determination is  $n$ , the radius of the probable error circle is  $P. E.$  Broadly speaking a determination of  $S_2$  must be considered unsatisfactory if  $A_2$  is less than three times  $P. E.$  It is reassuring to see that even for Isachsen, where the error circle is largest,  $P. E.$  is only 0.034 mb. when 5 years of data are used.

## 2. THE DATA AND THEIR ANALYSIS

Since the publication of Spar's paper, 3-hourly mean values of pressure and temperature, averaged for each month for the 10-yr. period 1951-1960, have been published for 40 Canadian stations by Cudbird [1]. These data have been subjected to harmonic analysis to

TABLE 1.— $S_2$  for six Canadian stations

Station	Lat. (° N.)	Long. (° W.)	$A_2$ (mb.)	$\alpha_2$ (°)	$P. E.$ (mb.)	$n$ (years)
Fort Smith.....	60.0	112.0	0.081	173	0.006	7
Whitehorse.....	60.7	135.1	.159	102	.019	9
Upper Frobisher.....	63.8	68.6	.164	199	.010	10
Cambridge Bay.....	69.1	105.1	.072	318	.005	7
Mould Bay.....	76.2	119.3	.077	327	.009	4.3
Isachsen.....	78.8	103.5	.097	279	.034	5.2

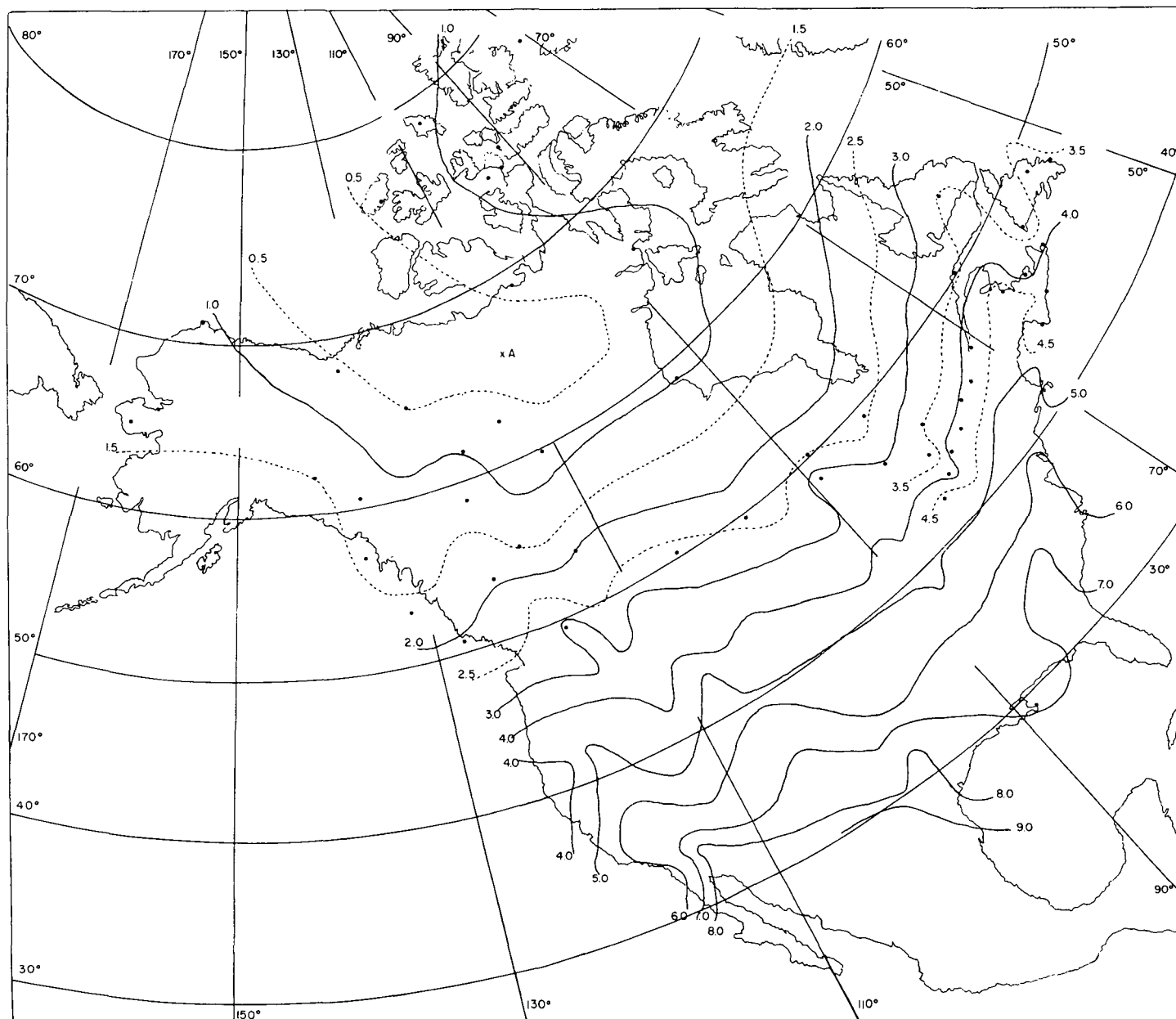


FIGURE 1.—The amplitude of  $S_2$  over North America (unit  $10^{-1}$  mb.). The dots indicate the available stations in Canada and Alaska. The location A in northern Canada denotes the amphidromic point. (United States data after Spar [7].)

For each of the 40 stations given by Cudbird the monthly and annual mean values of the harmonic coefficients for  $S_2$  were obtained, and from these the amplitudes and phase angles.

Next, maps of these amplitudes (fig. 1) and phase angles (fig. 2) were plotted for  $S_2(p)$ , the phase angles being expressed as that Local Mean Time when the pressure maximum occurs. It is not entirely satisfactory to separate the synoptic representations of amplitude and phase. But this procedure gives a clear picture of the spatial distribution of the two quantities. These maps of the distribution of  $S_2$  over Canada were then combined

with those published by Spar [7] for the United States. Very little difficulty was encountered in connecting these two areas since there was in general good agreement along the common boundary. In the vicinity of the Great Lakes the Canadian data seemed to justify some small alterations of Spar's original construction.

For determination of the monthly harmonic coefficients of  $S_2$ , a correction should be made because the mean pressure changes from month to month. However, the variation of the monthly mean pressure is so small that the correction is unnecessary. A further correction, applicable only to the phase constants of the monthly means

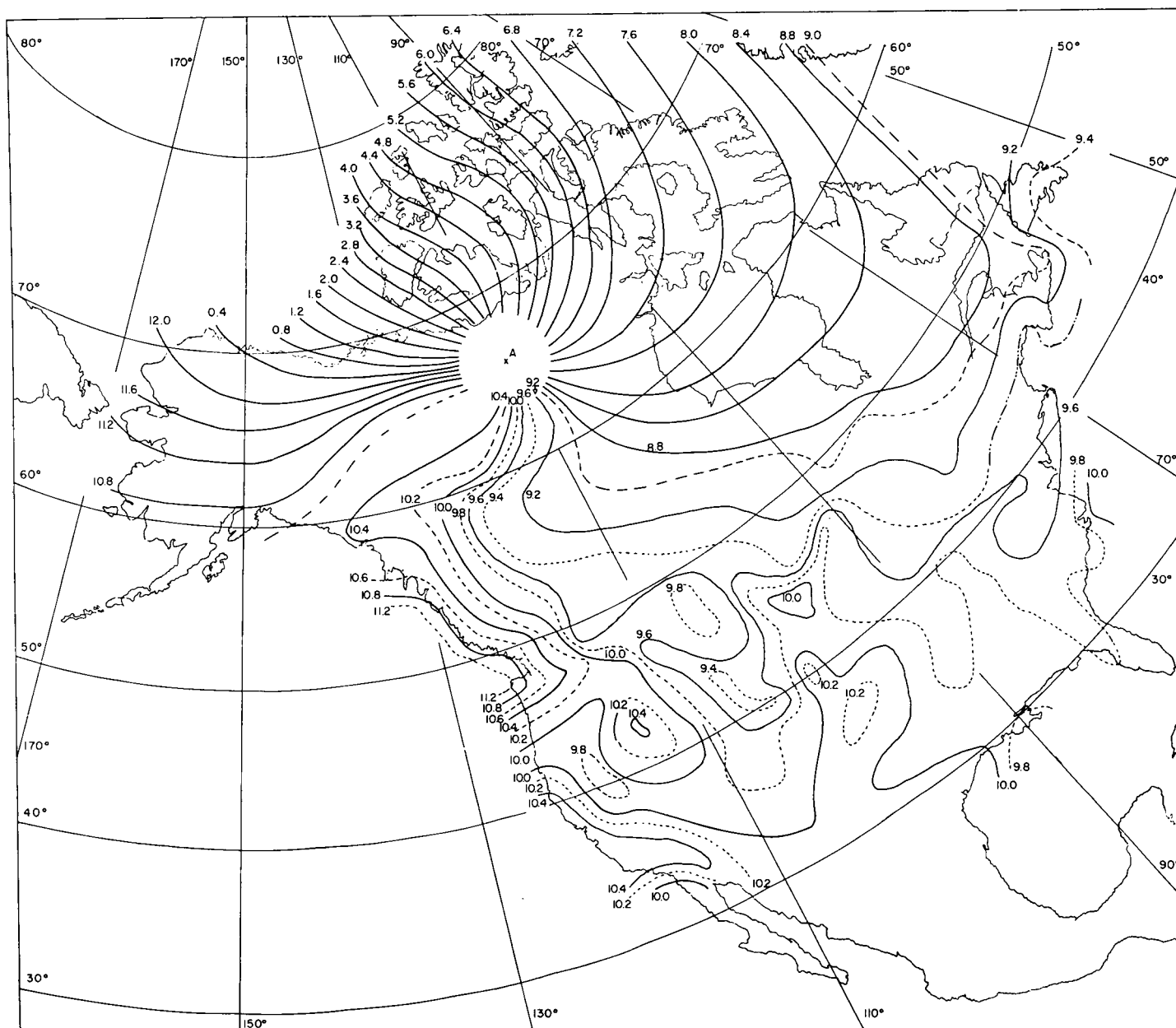


FIGURE 2.—The time of the pressure maximum of  $S_2$  (Local Mean Time). (United States data after Spar [7].)

of  $S_2$ , is that for the difference between Local Mean and True Solar Time, in other words, for the Equation of Time. This correction is easily applied to the phase expressed by the time when the maximum occurs.

In order to show the seasonal variation of  $S_2$  and its change with location the Canadian stations have been combined in three groups as follows:

(1) An *east coast group* comprising 21 stations from the Atlantic coast west to Windsor, Ontario ( $83^\circ$  W.). This group was at first subdivided into a western and eastern part. But since there is no appreciable difference between these two subdivisions they were combined into one group.

(2) A *central plains group* comprising 5 stations, the most westerly station being Edmonton, Alberta ( $113^\circ$  W.).

(3) A *western group* comprising 8 stations in the western mountains and on the west coast.

Some of the stations used in drawing figures 1 and 2 have not been included in these groups because they are too much under the influence of the standing oscillation.

The seasonal variations of  $S_2$  for these three groups are plotted in figure 3 in harmonic dials. This form of presentation is most appropriate when differences and variations in amplitude and phase are to be shown simultaneously.

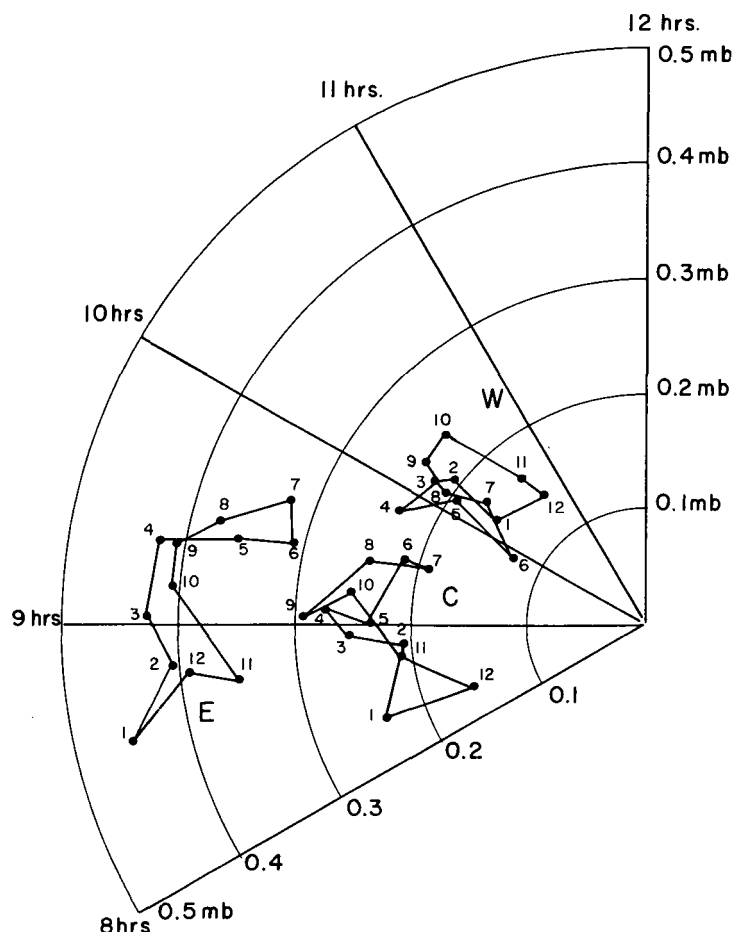


FIGURE 3.—Harmonic dial for the seasonal variation of  $S_2$  in Canada. E=eastern stations, C=central plains stations, W=western stations. The number at each point indicates the month.

### 3. DISCUSSION

#### THE ANNUAL MEAN OF $S$

The distributions of the amplitude and phase of  $S_2$  over North America are shown by figures 1 and 2, respectively. The area south of the boundary between Canada and the United States is, of course, a reproduction of Spar's [7] corresponding maps.

Figure 1 for the amplitude shows that in the southern part of the continent the lines of equal amplitude run largely east and west, decreasing northward in intensity, in agreement with the overall global distribution of  $S_2$ . Deviations from this simple picture are mainly confined to the western mountain area and the two coasts. Farther north the effect of the standing oscillation becomes more noticeable, and as a result the distribution of the amplitudes is also a function of geographic longitude. In particular, over the western part of the Northwest Territories of Canada a region exists of very low, and presumably even zero, amplitude, although no stations are available to verify observationally that the zero value is

reached. On theoretical grounds  $S_2$  must vanish at a point where the amplitudes of its two component vibrations, the traveling and standing waves, are equal, and where their phase constants differ by  $180^\circ$ .

It is possible to locate this point of vanishing  $S_2$  also by means of the phase distribution given in figure 2. In this figure are shown lines of equal Local Mean Time at which the maximum of  $S_2$  occurs. These lines of equal phase, which may also be called cotidal lines as in oceanography, are seen to converge into a point at  $66^\circ$  N.,  $111^\circ$  W., in the area of lowest amplitude. This point is referred to as the "amphidromic point" since the crest of the semidiurnal tide sweeps around it, once in 12 hr., similar to the amphidromic points in the oceans. In an earlier paper (Haurwitz [2]), the position of this point was computed to be at  $65.5^\circ$  N.,  $113.6^\circ$  W. (and at another point  $180^\circ$  of longitude distant) on the basis of a harmonic analysis of the worldwide distribution of  $S_2$ . The present determination of its position which is based on more (but not yet sufficient) stations is in good agreement with the earlier result.

Along the eastern seaboard of the northern United States and of southern Canada the maximum of  $S_2$  occurs earlier than over the central and western parts of the continent, but in general the distribution of the phase constant is much more complicated than that of the amplitude, especially over the United States. That the phase constant appears to be more regularly distributed over Canada is presumably mainly due to the sparsity of data available there. Along the west coast of the continent the maximum of  $S_2$  is considerably delayed compared to the rest of the continent. As suggested by Spar this retardation of the semidiurnal tide may be largely an orographic effect of the western mountains. The theory of such an orographic effect has been studied by Kertz [4]. But in the northern part of the continent this delay, as well as the early maximum in the eastern region, is at least partly also due to the standing oscillation which makes the phase constant of  $S_2$  dependent on the longitude.

#### SEASONAL VARIATION OF $S_2$

The seasonal variations for the three groups of Canadian stations are plotted in figure 3 in a harmonic dial. The three closed curves show the monthly march of  $S_2$  for the east coast group (E) the central plains group (C), and the western group (W), the numerals at each point representing the month. In general, the amplitudes are largest at the times of the equinoxes, small around the solstices, although the actual extremes may be delayed a month. The east coast stations have the greatest amplitude maximum in January, similar to Spar's findings for the east coast stations in the United States. The winter minimum for the eastern Canadian stations occurs in November, rather than in December.

The seasonal phase shift is large for the east coast and central plains stations with the pressure maximum occurring somewhat more than one hour earlier in midwinter

than in midsummer. The total seasonal phase variation for the west coast stations is somewhat less than an hour, with the earliest phase in April through June, the latest phase in October through December, very different from the conditions for the east coast and central plains groups. During every month the maximum occurs later at the western stations than over the central plains or the east, indicating that  $S_2$  is delayed as it approaches the western stations. In this context it is pertinent to recall again that Kertz's theoretical study of the orographic effects on  $S_2$  showed that the maximum should be delayed west of a mountain range.

From figure 3 it can be seen that the time difference between the maxima for the western group and the two other groups is largest in winter and smallest in summer. A similar variation of the phase difference throughout the year was found by Spar between the west coast and east coast stations in the United States.

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